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PROPAGATION OF A PLANE TWO-PHASE ELECTROHYDRODYNAMIC JET WITH LOW MOBILITY OF CHARGED PARTICLES IN A HOMOGENEOUS TURBULENT FLOW

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On the basis of several simplifying assumptions a system of equations is written for a two-phase EHD (electrohydrodynamic) jet of low mobility particles formed by a two-dimensional source of the charged aerosol in a homogeneous turbulent flow. This system is solved by the method of series expansion in a small parameter. The case of sufficiently large Reynolds numbers is considered when the restriction to the zero approximation is possible. A nonstationary bipolar jet and a stationary unipolar jet are examined.

1. Basic equations. We consider a turbulent EHD aerosol jet formed by a plane source of the charged aerosol which is placed in a homogeneous turbulent incompressible gas flow. We assume that the space charge density of the particles and their volume concentration are sufficiently small and therefore we can neglect the reaction of the particles on the turbulence of the flow and its averaged velocity. The particles are spherical and obey the Stokes drag law. The mobility of the particles is determined by the relationship $k = z / (6\pi\mu r)$ (Z, r are the charge and the radius of the particle, respectively, μ is the gas viscosity coefficient). In the general case charges of opposite signs can be present in the jet and $Z^+ = |Z^-| = \text{const}$. Coagulation and evaporation of the particles are not considered. The system of equations of electrohydrodynamics describes the true motion of the aerosol phase and coincides in this case with the conventional equations of electrohydrodynamics for a small interaction parameter [1, 2]

$$\partial q^\pm / \partial t \pm k^\pm \nabla \mathbf{E} q^\pm + \mathbf{u} \nabla q^\pm = 0, \qquad \nabla^2 \varphi = -\frac{1}{\varepsilon} q \tag{1.1}$$
$$q^\pm = Z^\pm N, \quad q = q^+ - q^-, \quad \mathbf{E} = -\nabla\varphi$$

Here $q^\pm$ is the space charge density of the particles, $N(t, x, y)$ is the number of particles in the volume unit, $\mathbf{E}$ is the electric field strength, $\mathbf{u}$ is the velocity of the gas flow, t is the time, ε is the dielectric constant of the gas.

Let us assume that in a turbulent flow the true values can be represented in the form of the sum of components averaged in time (marked by angular brackets) and pulsating components (marked by dashes). Averaging the equations (1.1) in the conventional manner [3], we obtain the following system of equations for a turbulent EHD flow:

$$\frac{\partial \langle q^\pm \rangle}{\partial t} \pm k^\pm \nabla (\langle \mathbf{E} \rangle \langle q^\pm \rangle) + \langle \mathbf{u} \rangle \nabla \langle q^\pm \rangle \mp \tag{1.2}$$
$$k^\pm \nabla \langle \mathbf{E}' q_\pm' \rangle + \nabla \langle \mathbf{u}' q_\pm' \rangle = 0, \qquad \nabla^2 \langle \varphi \rangle = -\frac{1}{\varepsilon} \langle q \rangle$$

The last two terms in the first equation of (1.2) describe a turbulent transfer of charged particles which is conditioned by a correlation between the pulsations of the space charge and pulsations of the electric field or of the flow velocity, respectively.

Assuming that the scale of the translating vortices in a homogeneous turbulent flow is small and $|\mathbf{u}'| \ll \langle \mathbf{u} \rangle$, $|q'| \ll |\langle q^{\pm} \rangle|$ [3, 4], we estimate the order of pulsation of the field $\mathbf{E}'$. For a jet propagating in the region S which is bounded by a grounded grid electrode not-distorting the hydrodynamic characteristics of the flow (in a particular case S represents an infinite strip), we can write the solution of Poisson's equation (1.1) in the form

$$\langle \mathbf{E} \rangle = \frac{1}{\varepsilon} \int_S \langle q \rangle \nabla G \, ds, \quad \mathbf{E}' = \frac{1}{\varepsilon} \int_S q' \nabla G \, ds, \quad \mathbf{E}' = -\nabla \varphi'$$

(here G is Green's function) from which follows that $|\mathbf{E}'| \ll |\langle \mathbf{E} \rangle|$ for $|q'| \ll |\langle q \rangle|$. If $k|\langle E_x \rangle| < \langle u_x \rangle$, $k|\langle E_y \rangle \langle q^{\pm} \rangle| \sim |\langle u_y' q_{\pm}' \rangle|$, and this is fulfilled for a sufficiently small mobility or density of the space charge in the jet, then, taking into account the relation between the scales of the fields $\langle \mathbf{E} \rangle$ and $\mathbf{E}'$ we obtain the estimates

$$k|\langle E_y' q_{\pm}' \rangle| \ll |\langle u_y' q_{\pm}' \rangle| \sim k|\langle E_y \rangle \langle q^{\pm} \rangle|, \quad \langle E_x' q_{\pm}' \rangle \ll \langle E_x \rangle \langle q^{\pm} \rangle$$

We assume that the last two conditions are valid and in (1.2) we neglect the term $k^{\pm} \nabla \langle \mathbf{E}' q_{+}' \rangle$. For a small volume concentration of the aerosol particles the term $\nabla \langle \mathbf{u}' q_{+}' \rangle$ in (1.2) can be represented in the form [3, 4]

$$\nabla \langle u' q_{+}' \rangle = -\nabla (D \nabla \langle q^{\pm} \rangle) \tag{1.3}$$

Here D is the coefficient of turbulent diffusion of the particles and represents either a scalar or a second rank tensor.

Without introducing great error in the case of the diffusion of particles in a homogeneous gas flow, we can neglect the turbulent transfer of particles in the direction of the mean velocity of the flow (henceforth the velocity $\langle u \rangle = \text{const}$ is directed along the x-axis) and take into account only the transverse diffusion, assuming $D = \text{const}$ [3]. For a nonstationary EHD flow the characteristic time T must be sufficiently large so that the diffusion process can be considered as a quasi-stationary. For a EHD jet propagating in an inhomogeneous gas flow, the coefficient of diffusion can be determined on the basis of semi-empirical theories of turbulent diffusion [3].

We introduce the nondimensional values

$$\begin{gathered} x_* = x / L, \quad y_* = y / L, \quad t_* = t / T, \quad \langle q_*^{\pm} \rangle = \langle q^{\pm} \rangle / A \\ \langle \varphi_* \rangle = \varepsilon \langle \varphi \rangle / AL^2, \quad \langle \mathbf{E}_* \rangle = \varepsilon \langle \mathbf{E} \rangle / AL, \quad k_*^{\pm} = k^{\pm} / k \\ H = L / T \langle u \rangle, \quad R = \varepsilon \langle u \rangle / kAL, \quad \mathrm{Pe} = \langle u \rangle L / D \end{gathered} \tag{1.4}$$

Then, taking into account the previous assumptions, the system of equations (1.2) can be written in the form

$$\begin{gathered} H \frac{\partial q^{\pm}}{\partial t} + \frac{\partial q^{\pm}}{\partial x} \mp R^{-1} \left(\frac{\partial \varphi}{\partial x} \frac{\partial q^{\pm}}{\partial x} + \frac{\partial \varphi}{\partial y} \frac{\partial q^{\pm}}{\partial y} - q^{\pm 2} \right) - \\ \mathrm{Pe}^{-1} \frac{\partial^2 q^{\pm}}{\partial y^2} = 0, \quad \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = -q \end{gathered} \tag{1.5}$$

(henceforth the asterisk for dimensionless values and the averaging sign are omitted). Here H is the parameter characterizing the nonstationary state of the flow, R is the electric Reynold's number, Pe is the Péclet number. For a periodic variation of the space charge density with time $H = \omega L / u$ ($\omega = 2\pi\nu$, ν is the angular frequency).

We note that the system of equations (1.5) is also valid in the case of an EHD jet of weakly charged gas ions, if the assumptions mentioned above are fulfilled. The equations describing the propagation of the ion laminar EHD jet, are determined in [2], where is shown that the molecule diffusion of ions can be neglected for $R \sim 1$. For a turbulent EHD jet $D \sim 10^{-2}$ m^2/sec, $\mathrm{Pe} \sim 10^4$ at $u \sim 10^{-2}$ m/sec, $L \sim 1$ m. Assuming $q^\pm \sim 1$, $x \sim 1$, $y \sim b/L \sim 10^{-2}$ (b is the characteristic transverse dimension of the EHD jet), we obtain $\partial q^\pm/\partial x \sim 1$, $\mathrm{Pe}^{-1}\partial^2 q^\pm/\partial y^2 \sim L^2/b^2\mathrm{Pe} \sim 1$. It follows from the estimates quoted that when examining "narrow" turbulent jets $(b/L \ll 1)$, it is necessary to take into account the turbulent transfer of space charge. In the case of "wide" jets $(b/L \sim 1)$ the influence of the turbulence can be neglected if $R^{-1}\partial(E_y\ q^\pm)/\partial y \sim 1$. For aerosol EHD jets in many cases of practical importance $R \gg 1$. For example, for $r \sim 10^{-5}$ m, $Z \sim 10^{-14}$ c, $N \sim 10^9$ m^{-3}, $u \sim 10^2$ m/sec, $\mu \sim 2\cdot 10^{-5}$ n sec/m^2, $\varepsilon \sim 10^{-11}$ f/m, $L \sim 1$ m, we obtain $A \sim 10^{-5}$ c/m^3, $k \sim 10^{-6}$ m^2/V. sec, $R \sim 10^2$.

Due to circumstances indicated we can solve the system (1.5) by the method of series expansions with respect to the small parameter R^{-1} assuming

$$q^\pm = \sum_{m=0}^{\infty} q^\pm_{(m)} R^{-m}, \qquad \varphi = \sum_{m=0}^{\infty} \varphi_{(m)} R^{-m} \tag{1.6}$$

Substituting (1.6) into (1.5) and equating terms of the same power of R, we obtain the following recurrent system:

$$H\frac{\partial q^\pm_{(0)}}{\partial t} + \frac{\partial q^\pm_{(0)}}{\partial x} - \mathrm{Pe}^{-1}\frac{\partial^2 q^\pm_{(0)}}{\partial y^2} = 0 \tag{1.7}$$

$$H\frac{\partial q^\pm_{(m)}}{\partial t} + \frac{\partial q^\pm_{(m)}}{\partial x} - \mathrm{Pe}^{-1}\frac{\partial^2 q^\pm_{(m)}}{\partial y^2} = \frac{\partial F_x^{(m-1)}}{\partial x} + \frac{\partial F_y^{(m-1)}}{\partial y} \qquad (m \geqslant 1)$$

$$\frac{\partial^2 \varphi_{(m)}}{\partial x^2} + \frac{\partial^2 \varphi_{(m)}}{\partial y^2} = -\,q_{(m)} \qquad (m \geqslant 0)$$

$$F_z^{(m-1)} = \frac{\partial \varphi_{(0)}}{\partial z}\, q^\pm_{(m-1)} + \frac{\partial \varphi_{(1)}}{\partial z}\, q^\pm_{(m-2)} + \ldots + \frac{\partial \varphi_{(m-1)}}{\partial z}\, q^\pm_{(0)} \qquad (z = x,\ y)$$

As a result the problem at each stage is reduced to the system of two linear equations of the second order: a parabolic one for $q^\pm_{(m)}$ and an elliptic one for $\varphi_{(m)}$. If the symmetric EHD jet propagates in the gap between the two infinite earthed electrodes in the form of grids *1*, *2* placed at the cross sections $x = 0$ and $x = 1$ (Fig. 1), then the

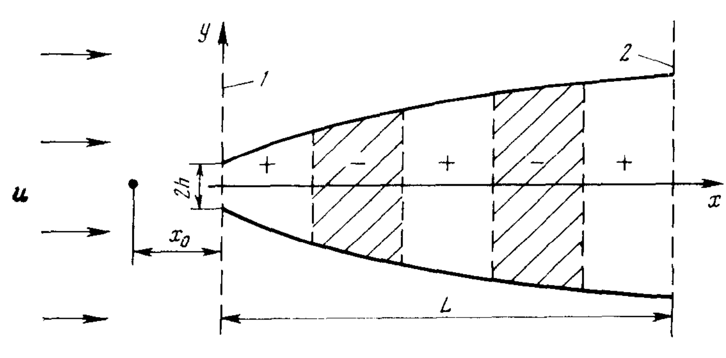

Fig. 1

initial and boundary conditions have the form

$$q_{(0)}^{+} = q_0^{+}(t, y), \quad q_{(0)}^{-} = q_0^{-}(t, y), \quad \varphi_{(0)} = 0 \tag{1.8}$$
$$q_{(m)}^{+} = q_{(m)}^{-} = \varphi_{(m)} = 0 \quad (x = 0,\ m \geqslant 1)$$
$$\varphi_{(m)} = 0 \quad (x = 1,\ m \geqslant 0)$$
$$\frac{\partial q_{(m)}^{\pm}}{\partial y} = \frac{\partial \varphi_{(m)}}{\partial y} = 0 \quad (y = 0,\ m \geqslant 0)$$
$$q_{(m)}^{\pm} \to 0, \quad \varphi_{(m)} \to 0 \quad (y \to \infty,\ m \geqslant 0)$$

Here $q_0^{\pm}$ is the dimensionless space charge density of the corresponding sign of the section edge of the source of the ionized particles.

We consider next symmetrical EHD jets spreading in the stream with homogeneous and isotropic turbulence for $R^{-1} \ll 1$ taking into account only the zero approximation. We assume that the grid electrodes do not distort that characteristics of the gas flow.

2. The propagation of a nonstationary periodic bipolar EHD jet. We introduce a new dependent variable $Q = q_{(0)}^{+} - q_{(0)}^{-}$. In this case the system of equations for the zero approximation and for the boundary conditions for Q have the form

$$H\frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial x} - \mathrm{Pe}^{-1}\frac{\partial^2 Q}{\partial y^2} = 0, \quad \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = -Q \tag{2.1}$$

$$Q = f(t)Q_0(y) \ \ (x = 0); \ \ \partial Q/\partial y = 0 \ \ (y = 0); \ Q \to 0 \ (y \to \infty) \tag{2.2}$$

Here and hereafter the subscript of φ is omitted. In writing down (2.2) a particular form of time dependence of the space charge density was chosen for the section edge of the source $q_{(0)}^{\pm} = f(t)q_{(0)}^{\pm}(y)$.

We note that in Eq. (2.1) the absence of the parameters depending on the mobility of the particles permits the causes of time variations of the space charge density on the section edge of the source of the particles to be not specified. The charge density can change because of the change of N for $Z^{\pm} = \mathrm{const}$ and also because of the change of $Z^{\pm}$ for $N = \mathrm{const}$.

Solving Eqs. (2.1) we apply the Fourier cosine transformation

$$F^*(\tau, x, t) = \sqrt{\frac{2}{\pi}}\int_0^\infty F(x, y, t)\cos\tau y\, dy \tag{2.3}$$

After transforming (taking into account (1.8), (2.2)) we obtain the system, the solution of which has the form

$$Q^* = Q_0^*(\tau)f(t - Hx)\exp(-\tau^2 x/\mathrm{Pe}) \tag{2.4}$$
$$\varphi^* = \frac{2}{\pi}\int_0^\infty \frac{\cos\tau y}{\tau^2\operatorname{sh}\tau}\Big[\operatorname{sh}\tau x\int_0^1 Q^*(\eta)\operatorname{sh}\tau(1-\eta)\,d\eta -$$
$$\operatorname{sh}\tau\int_0^x Q^*(\eta)\operatorname{sh}\tau(x-\eta)\,d\eta\Big]d\tau$$

Let us consider a particular case in which a source of a finite width $2h$ periodically generates space charge accordingly to the law

$$Q_0(t) = \beta\cos t, \quad x = 0, \quad \beta = \begin{cases} 1, & |y| < h \\ 0, & |y| > h \end{cases} \tag{2.5}$$

As the characteristic density of the space charge A, the time T and the dimension L for the periodic EHD jets, it is convenient to choose $A = \max|q_0^+|$ (here $\max q_0^+ = \max|q_0^-|$, q_0^+, q_0^- are the dimensional values), $T = 1/\omega$, $L = u/\omega$, and $\max|Q_0| = 1$, $H = 1$, $\mathrm{Pe} = u^2/\omega D$. The total charge generated by the source during one period is equal to zero, therefore in forming a jet it is not necessary to set a collector-electrode 2 (Fig. 1) at a finite distance from the source or to compensate the volume charge in some other manner. Assuming that the collector-electrode is placed at infinity and $\varphi|\xrightarrow[x\to\infty]{} 0$, we obtain on the basis of (2.4), (2.5), (2.3)

$$Q = \frac{1}{2}\cos(t-x)\left[\Phi\left(\frac{(y+h)}{2}\sqrt{\frac{\mathrm{Pe}}{x}}\right) - \Phi\left(\frac{(y-h)}{2}\sqrt{\frac{\mathrm{Pe}}{x}}\right)\right]$$

$$\varphi = \frac{1}{\pi}\int\limits_0^\infty \frac{e^{-\tau x}\cos\tau y\sin h\tau}{\tau^2\Omega_1\Omega_2}\{\Omega_1[\sin(t+\alpha_1) - e^{-\Omega}\sin(t-x+\alpha_1)] -$$

$$\Omega_2[\sin(t+\alpha_2) - e^{-\Omega}\sin(t-x+\alpha_2)]\}\,d\tau$$

$$\Omega = \tau x\left(1+\frac{\tau}{\mathrm{Pe}}\right), \quad \Omega_{1,2} = \sqrt{\tau^2\left(\frac{\tau}{\mathrm{Pe}}\pm 1\right)+1}$$

$$\alpha_{1,2} = \arcsin\left[1+\tau^{-2}\left(1+\frac{\tau}{\mathrm{Pe}}\right)^{-2}\right]^{-1/2}$$

The character of the variation of the longitudinal field E_x and the space charge density Q along the axis of the jet at $\mathrm{Pe} = 500$, $t = 0$, is shown in Fig. 2. The cross sections $x = \pi m/2$ $(m = 1,2, \ldots)$ represent the boundaries of the regions of a mononomical charge. For $x \geqslant \pi$ the extremes of the field E_x are situated in the points $x = \pi m/2$. For $x \sim 4\pi\cdot 10^2$ the field is practically equal to zero.

3. The stationary unipolar EHD jet. Let us consider a stationary unipolar EHD jet formed by a linear source of ionized aerosol particles and situated at a distance x_0 ahead of the electrode 1 (Fig. 1). The initial system of equations has the form (2.1) with $H = 0$, $Q = q$. As the characteristic density of the space charge in (1.4) the value $A = I/uL$ is chosen, I is the current carried by the jet. The equation of current continuity in the case considered agrees with the equation of thermal conduction, the solution of which is:

$$q = \frac{1}{2}\sqrt{\frac{\mathrm{Pe}}{\pi(x+x_0)}}\exp\left[-\frac{y^2\,\mathrm{Pe}}{4(x+x_0)}\right] \tag{3.1}$$

for the condition $q_0(-x_0, 0) = \delta(y)$ (δ is the delta dunction). Accordind to the method of Grinberg [5], we represent the solution of Poisson's equation in the series form

$$\varphi = \sum_{n=1}^\infty U_n\sin\pi nx, \quad U_n = 2\int\limits_0^1\varphi(x, y)\sin\pi nx\,dx \tag{3.2}$$

Multiplying the second equation of (2.1) by $\sin\pi nx$ and integrating it in accordance with (3.2), we obtain an ordinary differential equation

$$\frac{d^2U_n}{dy^2} - \pi^2n^2U_n + q^* = 0, \quad q^* = 2\int\limits_0^1 q\sin\pi nx\,dx \tag{3.3}$$

with the boundary conditions $dU_n/dy = 0$ $(y = 0)$, $U_n \to 0$ $(y \to \infty)$. Taking into account (3.1), (3.2) and integrating (3.3) we obtain the final expression for the potential

$$\varphi = \frac{1}{2\pi}\sum_{n=1}^{\infty}\frac{\sin \pi n x}{n}\int_0^1 \sin \pi n \eta \exp\left[\pi^2 n^2 (\eta + x_0)/\,\mathrm{Pe} - \pi n y\right] \times \tag{3.4}$$
$$\{1 - \Phi(z_1) + [1 - \Phi(z_2)]\exp(2\pi n y)\}\, d\eta$$
$$z_{1,2} = \pi n \sqrt{\frac{\eta + x_0}{\mathrm{Pe}}} \mp \frac{y}{2}\sqrt{\frac{\mathrm{Pe}}{\eta + x_0}}$$

where ($\Phi(Z)$ is the probability integral. On the basis of (3.4) φ, E_x, E_y were computed numerically for $x_0 = 10^{-3}$, $\mathrm{Pe} = 10^4$, 10^5, 10^6 which correspond to the inter-electrode gap lengths $L = 1, 10, 100$ m and $b(0) = 1.4, 4.3, 13.5$ mm at $u = 10^2$ m/sec, $D = 10^{-2}$ m^2/sec. The half-width of the EHD jet $b(x)$ has been determined from the condition $q(x,b)/q(x,0) = 0.01$ and Eq. (3.1)

$$b = 3.03\sqrt{2(x + x_0)/\mathrm{Pe}}$$

We note that "narrow" jets of large length ($b/L \ll 1$, $L \sim 10$ to 10^2 m) can occur, e. g. during the operation of an aerosol EHD neutral agent of static electricity in the flight of an aircraft.

The distribution of the potential along the axis of the jet for different Re is shown in Fig. 3 (the curves *1*—*3* correspond to $\mathrm{Pe} = 10^4$, 10^5, 10^6). The maximum of potential is near $x = 0.5$; variation in Pe results in small changes of $\varphi(x, 0)$. For $y > b$ a shift of the maximum occurs towards the electrode *1* and a decrease of the potential with increase of y ($\mathrm{Pe} = 10^6$, in Fig. 3 the curves *4*—*6* correspond to the values of $y = 0.1$, 0.5, 1).

The variation of the potential across the jet for $\mathrm{Pe} = 10^4$ (solid lines) and $\mathrm{Pe} = 10^6$ (dashed lines) is shown in Fig. 4 (the curves *1*, *2*, *3* correspond to the cross sections $x = 0.2$, 0.5, 0.7). For $\mathrm{Pe} \sim 10^6$ the potential along the cross section is practically constant ($0 \leqslant y / b \leqslant 1$). For relatively wide jets ($\mathrm{Pe} \sim 10^4$) the potential attains its maximum on the axis of the jet and diminishes as the boundary $y = b$ is approached. In this case the character of the potential variation coincides qualitatively with the results in [6] of the numerical integration of the equations for an EHD jet of ions at $R \sim 1$, $\mathrm{Pe} \to \infty$, $b / L \sim 1$. Outside the jet (left part of Fig. 4) the potential diminishes monotonically with the increase of y and at $y \sim 1.5$ is practically equal to zero. This indicates the possibility of an approximate replacement of the inter-electrode gap in the form of an infinite strip of a unit width by a rectangle with sides $x = 1$, $y = 1.5$; this can be applied for numerical computations of the EHD jets for $\mathrm{Pe} \geqslant 10^4$.

The electric field strength E_x on the jet's axis varies nearly linearly for $0.3 < x < 0.7$ and increases sharply at small distances from the electrodes; $|E_x(0,0)| > E_x(1,0)$ (see the curves *1*—*3* in Fig. 5 which correspond to $\mathrm{Pe} = 10^4$, 10^5, 10^6). The longitudinal field varies a little across the jet for $\mathrm{Pe} \sim 10^6$ (see Fig. 6, where $E_x^* = E_x / E_x(x, 0)$, $\mathrm{Pe} = 10^6$, the curves *1*—*3* correspond to $x = 0$, 0.5, 1), therefore for the narrow jets we can assume approximately that $E_x \approx f_1(x)$ for $0 \leqslant y \leqslant b$. The character of the variation of E_x outside the jet is shown by curves *4*, *5*, *6* in Fig. 5 (respectively $y = 0.1$, 0.5, 1 for $\mathrm{Pe} = 10^6$).

The distribution of the transverse field E_y in the different cross sections of the jet for $\mathrm{Pe} = 10^6$ is shown in Fig. 7. The curve E_y is also shown (dotted); it is calculated on the basis of the approximation for the "boundary layer" [7, 8] under the condition

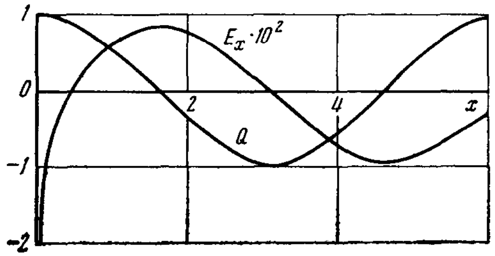

Fig. 2

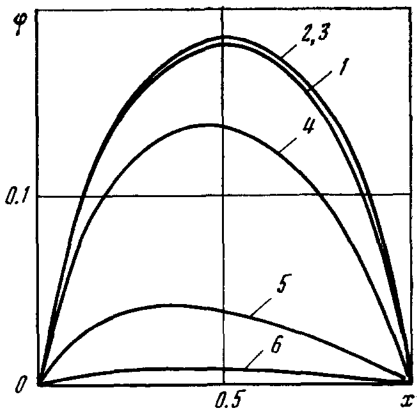

Fig. 3

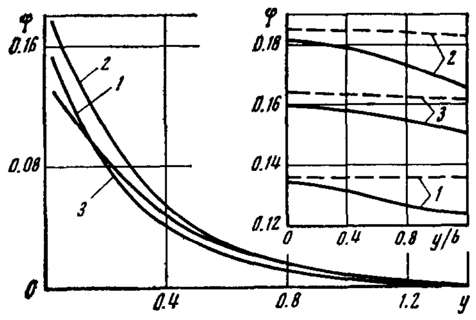

Fig. 4

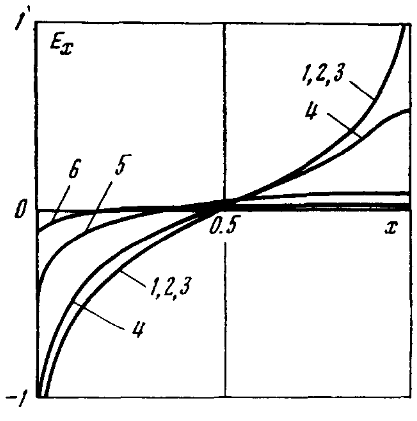

Fig. 5

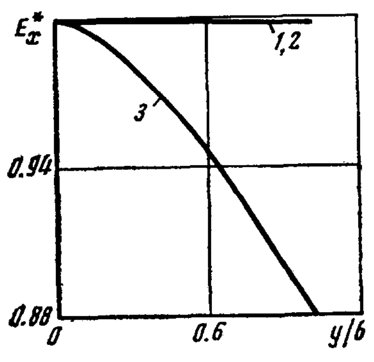

Fig. 6

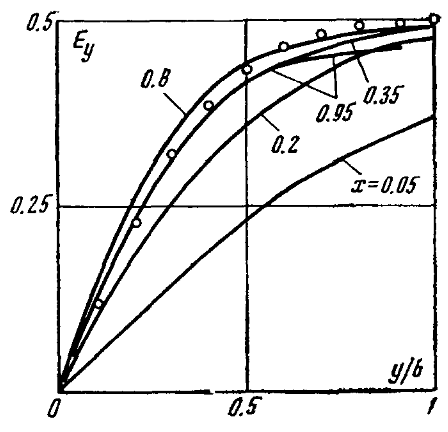

Fig. 7

$\partial E_x / \partial x \ll dE_y / \partial y$ (actually, this condition is satisfied only near the axis of the jet in the gap $0.2 < x < 0.8$). The solution of the simplified Poisson's equation $\partial E_y / \partial y = q$ for the symmetrical jet ($E_y(x, 0) = 0$), in the boundary layer approximation and (3.1), (3.5) taken into account, has the form

$$E_y = \frac{1}{2} \Phi \left(\frac{3.03}{b\sqrt{2}} y\right)$$

It follows from a comparison of the curves in Fig. 7 that the approximation of the boundary layer describes with an accuracy up to 10% the transverse component of the field for $0.35 \leqslant x \leqslant 0.95$, $0 \leqslant y \leqslant b$. The longitudinal field E_x within the limits of the indicated approximation is not determined and must be obtained on the basis of additional assumptions. In the gap $0 \leqslant y < \infty$ the transverse field E_y attains the maximum near the boundary of the jet for $y \geqslant b$ and then decreases monotonically with increase of y and for $y > 0.1$, $\mathrm{Pe} = 10^4$ to 10^6 its value is practically independent of Pe.

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